A Note to Revisiting Sequential Search Using Question-sets with Bounded Intersections

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The aim of this note is to explain why our paper solves a generalization of a well-known puzzle which is the following. We are given two ostrich eggs and a hundred story building and our goal is to determine (with as few trials as possible *in the worst case*) which is the lowest floor from where if we drop an egg, it breaks. Eg., if we had one egg, we would need ninety-nine trials since we cannot risk the egg to be broken and we assume that it always breaks if we drop it from the top floor. In the generalization the building has n floors and we have m eggs. We claim that this problem is equivalent to the following search problem. From a set of n elements we want to find (with as few questions as possible *in the worst case*) a given element using questions of the form "$x \in A$?" where x is the searched element and A is a subset with the additional constraint that the intersection of any m of these subsets that we ask is at most one. Now we show that these problems are indeed the same.

First we show that if the puzzle with the eggs can be solved using $f(n, m)$ drops, then we can also solve the search problem with $f(n, m)$ questions. Wlog. assume that the set is $\{1, \ldots, n\}$. If the first egg is dropped from the l^{th} level, then the first set that we ask is $\{1, \ldots, l\}$. If the searched element is not in the set, then we assume that the egg is not broken and continue with the remaining $n-l$ levels / set of size $n-l$. If the searched element is in the set, then we assume that the egg is broken and continue with one less egg in an l story building / with the restriction that the intersection of any $m-1$ sets is at most one in an l element set. At the end when we are left with only story, we will have a set of one element, the searched one, thus we are done. A similar reasoning shows that if we can solve the search problem with $f(n, m)$ questions, then we can also solve the puzzle with the eggs using $f(n, m)$ drops, hence they are indeed equivalent.

In the paper, this function is denoted by $f_1(n, m)$ (so this was the $k = 1$ special case of the problem investigated there) and its value is determined by Theorem 3.2 and Corollary 3.3, it is $(n \cdot m!)^{1/m} + m/2 + \Theta(1)$ if n is big enough.