Odd-Sunflowers

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Joint work with
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Sunflowers

Definition

A family of at least 3 sets is a sunflower (or a Δ-system) if every element is contained in no sets, in exactly 1 set, or in all of the sets.

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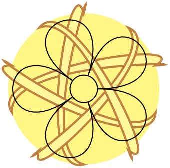

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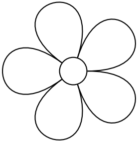

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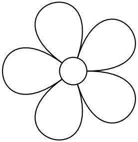

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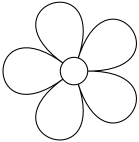

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Conjecture (Erdős, Rado 1960)

If k-uniform $\mathcal{F}$ is sunflower-free, $|\mathcal{F}| \leq C^k$.

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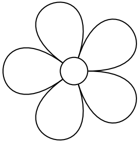

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Theorem (Alweiss, Lovett, Wu, Zhang '19)

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Odd-sunflowers and Even-sunflowers

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5. Some more:

1	2	3			
1	2		4		
1		3		5	
	2	3			6

1	2	3			
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		3	4	5	
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1				5	6
	2			5	6

Odd-town and Even-town

Classic applications of linear algebra in combinatorics.

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Theorem (Odd-town)

In a family $\mathcal{F}$ over $\{1,\ldots,n\}$ if every set is odd, but the intersection of any two sets is even, then $|\mathcal{F}| \leq n$.

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Does linalg work for odd- or even-sunflowers?

Even-sunflowers with linalg

Theorem (Even-sunflower-town)

If family $\mathcal{F}$ over $\{1, \ldots, n\}$ is even-sunflower-free, then $|\mathcal{F}| \leq n$.

Even-sunflower: each element is in an even number of sets.

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There are $2^{|\mathcal{F}|} - 1$ nonempty subfamilies of $\mathcal{F}$.
If $|\mathcal{F}| > n$, some two subfamilies contain same elements odd times.
Their symmetric difference is an even-sunflower. □

Odd-sunflowers with slicerank

Theorem (Odd-sunflower-town)

If family $\mathcal{F}$ over $\{1, \ldots, n\}$ is odd-sunflower-free, then $|\mathcal{F}| < 1.89^n$.

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Can we separate them/bound μ_{odd} without slicerank method?

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Theorem (Deuber, Erdős, Gunderson, Kostochka, Meyer 1997)

$\mu > 1.551$; *sunflower-free* $\mathcal{F}$ *over* $\{1, \ldots, n\}$ *with* $|\mathcal{F}| > 1.551^n$.

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Theorem (Frankl, Pach, P.)

$\mu_{odd} > 1.5$; *odd-sunflower-free* $\mathcal{F}$ *over* $\{1, \ldots, n\}$ *with* $|\mathcal{F}| > 1.5^n$.

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Odd-sunflower-free constructions

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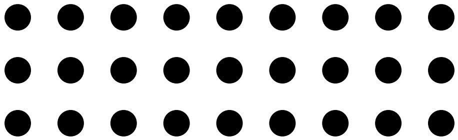
Take $3 \times \lfloor \frac{n}{3} \rfloor$ rectangle; sets have two from each column.



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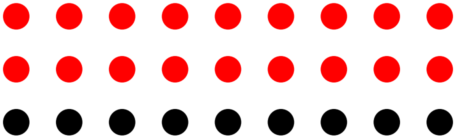

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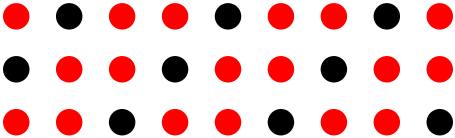

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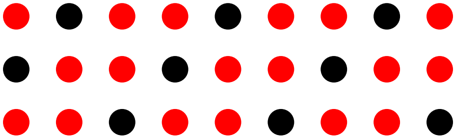

This gives $|\mathcal{F}| = 3^{n/3} > 1.44^n$.

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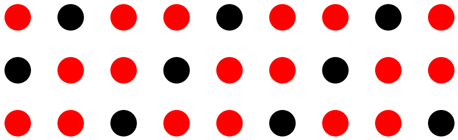

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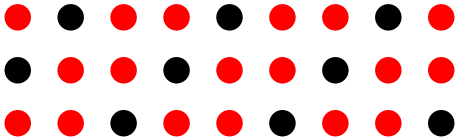

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Lemma

If $\mathcal{F}$ and $\mathcal{G}$ are odd-sunflower-free antichains, then so is $\mathcal{F} + \mathcal{G}$.

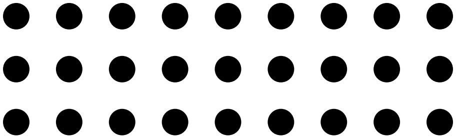
Definition: $\mathcal{F} + \mathcal{G} = \{F \dot\cup G \mid F \in \mathcal{F}, G \in \mathcal{G}\}$.

Odd-sunflower-free constructions

Construction (2)

Odd-sunflower-free $\mathcal{F}$ *over* $\{1, \ldots, n\}$ *with* $|\mathcal{F}| > 1.502144^n$.

Take 3×9 rectangle; sets have two from eight columns.



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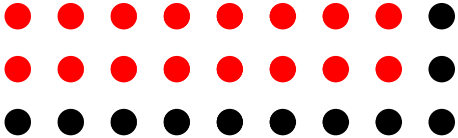

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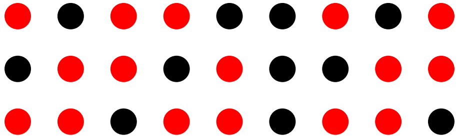

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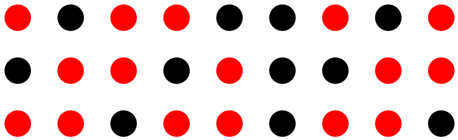

This gives $\mu_{odd} = (9 \cdot 3^8)^{1/27} = 3^{10/27} > 1.502144$.

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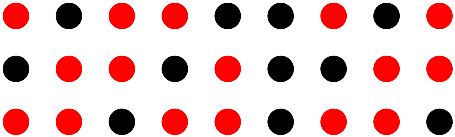

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$$\mathcal{F} \wr \mathcal{G} = \{\bigcup G_i \mid F \in \mathcal{F}, G_i \in \mathcal{G}_i \text{ for } i \in F\}.$$

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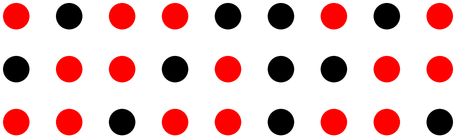

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$\binom{[n]}{n-1}$: All $n-1$ size sets of $\{1, \ldots, n\}$.

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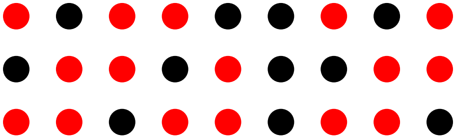

This gives $\mu_{odd} = (9 \cdot 3^8)^{1/27} = 3^{10/27} > 1.502144$.

Lemma

If $\mathcal{F}$ and $\mathcal{G}$ are odd-sunflower-free antichains, then so is $\mathcal{F} \wr \mathcal{G}$.

$$\mathcal{F} \wr \mathcal{G} = \{\bigcup G_i \mid F \in \mathcal{F}, G_i \in \mathcal{G}_i \text{ for } i \in F\}.$$

$\binom{[n]}{n-1}$: All $n-1$ size sets of $\{1,\dots,n\}$. Constr. (2) is $\binom{[9]}{8} \wr \binom{[3]}{2}$.

Odd-sunflower-free constructions

Construction (3)

Odd-sunflower-free $\mathcal{F}$ *over* $\{1, \ldots, n\}$ *with* $|\mathcal{F}| > 1.502148^n$.

Take $3 \times 9 \times 160511$ grid; sets have two from eight columns from one hundred sixty thousand five hundred ten horizontal sections.

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That is, $\mathcal{F} = \binom{[160511]}{160510} \wr \left(\binom{[9]}{8} \wr \binom{[3]}{2}\right)$.

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If $\mathcal{G}$ is an odd-sunflower-free antichain on m elements, then so is $\binom{[n]}{n-1} \wr \mathcal{F}$ on mn elements, with $n|\mathcal{G}|^{n-1}$ sets.

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Shows that no "additive" construction can reach the limit μ_{odd}.

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Thank you for your attention!