Unlabeled Compression Schemes

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MTA-ELTE CoGe, Budapest

28 June 2018

Learning theory

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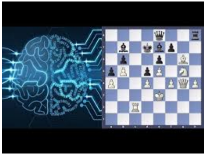

AlphaZero

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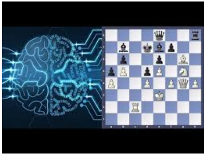

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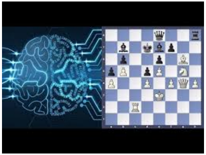

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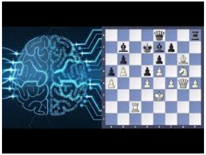

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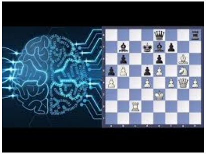

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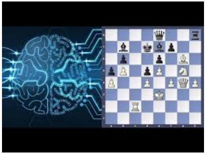

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Key to compress data — experts guess lookup table (oracle) for chess compressable to ≈300MB.

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Concept	*Sample*	*Compressed sample*	*Decoding*
$F \in \mathcal{F}$	S	$\alpha(S)$	$\beta(\alpha(S))$
$\pm_F$	mask of F	mask of S	$\notin \mathcal{F}$ allowed
$\{+,-\}$'s	$\{+,-,.\}$'s	$\{+,-,.\}$'s	$\{+,-\}$'s

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Compression scheme is valid if $\beta(\alpha(S))$ matches S wherever $S \neq .$

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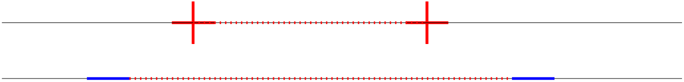

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By definition $\mathrm{LCS} \leq \mathrm{UCS}$.

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Proof. Take $|S| = d$.
There are 2^d labelings of S and 2^d (unlabeled) subsets of S. □

VC $\not\leq$ LCS

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Sample	*Compression*	*Decoding*
$\{+,+\}$	$\{+,.\}$	$\{+,+\}$
$\{-,-\}$	$\{-,.\}$	$\{-,-\}$
$\{+,-\}$	$\{.,-\}$	$\{+,-\}$
$\{-,+\}$	$\{.,+\}$	$\{-,+\}$
$\{+,.\}$	$\{+,.\}$	$\{+,+\}$
$\{.,-\}$	$\{.,-\}$	$\{+,-\}$
$\emptyset$	$\emptyset$	any

$\mathrm{VC} \leq 5 \cdot \mathrm{LCS}$

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2^d labelings of S, and labeled subsets of S of size $\leq d/5 =$

$$\sum_{i=0}^{d/5} \binom{d}{i} 2^i \leq \left(\frac{ed}{d/5}\right)^{d/5} 2^{d/5} \leq (10e)^{d/5} < 2^d. \quad \square$$

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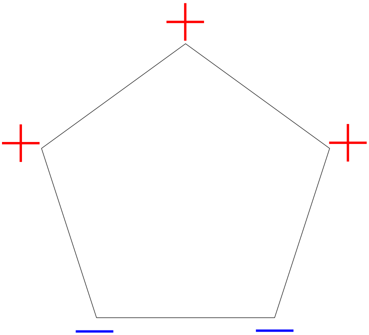
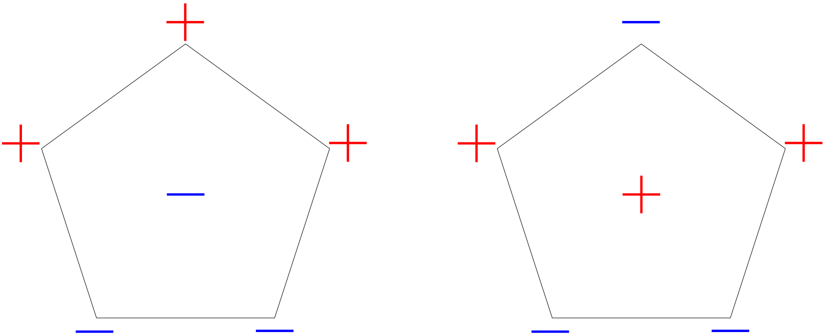
$|C_5| = 10$, the 5 rotations of the following sets:



UCS $\not\leq$ VC

Family C_5 can be made more symmetric by extending onto one more element (still $\mathrm{VC} = 2$, $\mathrm{UCS} = 3$, $|W_6| = 10$), 5 rotations of:



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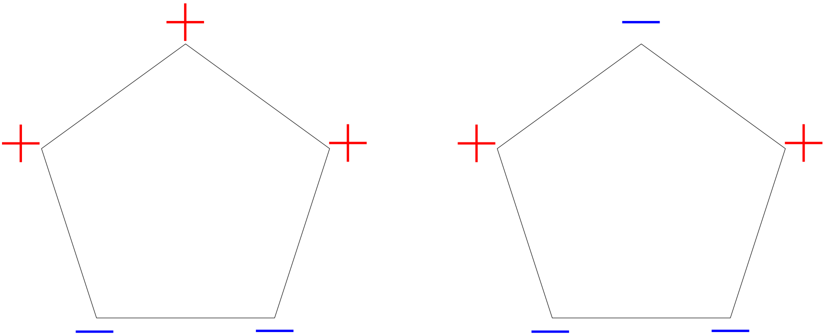

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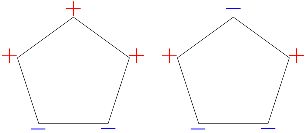

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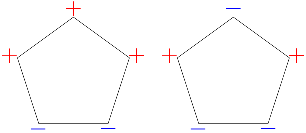

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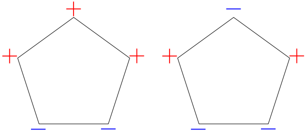

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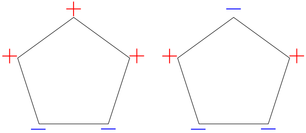

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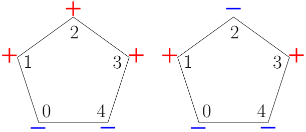

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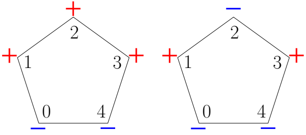

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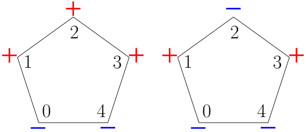

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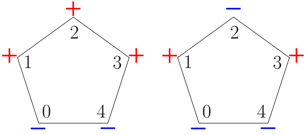

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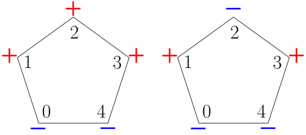

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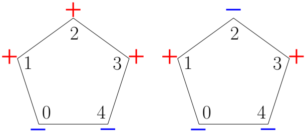

One of these must occur somewhere. □

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Sample	*Compression*	*Decoding*
no triples	mark all $+$'s	marks $+$, rest -
triple $+$ in $\mathcal{F}$	mark triple and $+$'s in $\mathcal{G}$	triple from position, marks in $\mathcal{G}$ same
triple - in $\mathcal{F}$	mark triple and -'s in $\mathcal{G}$	
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Open. Is $\mathrm{UCS}(\mathcal{F} * [n]) = \mathrm{VC}(\mathcal{F} * [n])$ for large enough $n = n(\mathcal{F})$?

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Open. Is $\mathrm{O1G}(\mathcal{F}) = O(\mathrm{VC}(\mathcal{F}))$ for finite $\mathcal{F}$?

Thank you for your attention!