Advantage in the discrete Voronoi game

8^{th} Japanese-Hungarian Symposium

Dömötör Pálvölgyi

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Joint work with:

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Dániel Gerbner,

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?????

Dedicated to

Szegő Laci
1972-2013

I have found another photo next to the previous one...

I have found another photo next to the previous one...

Guess who this is!

I have found another photo next to the previous one...

Guess who this is!

Wiener Gábor

What is the Voronoi game?

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App by Indrit Selimi!

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So First and Second claim points alternating for t rounds.

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So First and Second claim points alternating for t rounds.

At end area is divided, each point goes to closest claimed.

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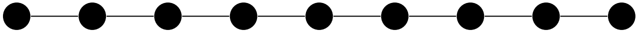

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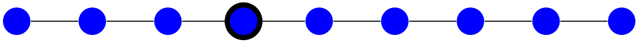

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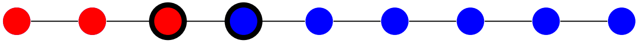

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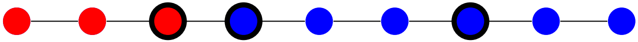

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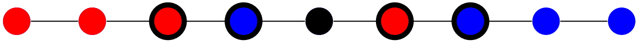

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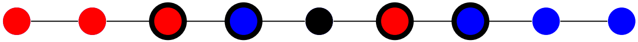

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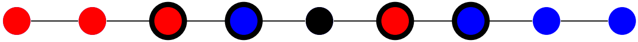

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Theorem (Kiyomi, Saitoh, Uehara)

Game on path is a draw unless odd vertices and $t = 1$.

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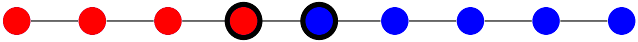

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Moreover, even then First wins with only one.

What percentage can each player get?

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Definition

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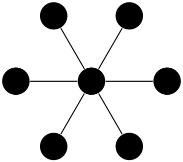

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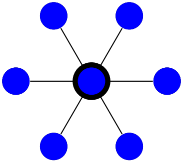

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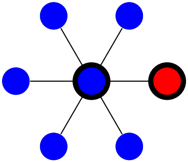

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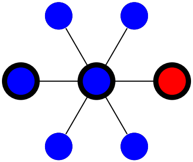

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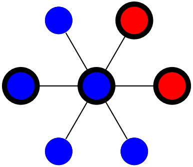

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Claim

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Proof.

First takes center of tree. □

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What if $t = 1$?

Examples

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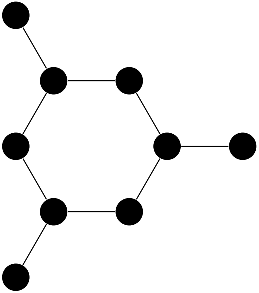
$$VG(G,1) = \frac{4}{9}$$



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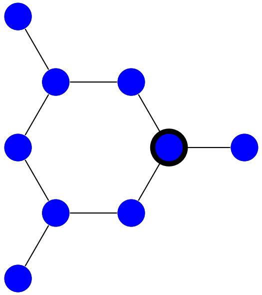

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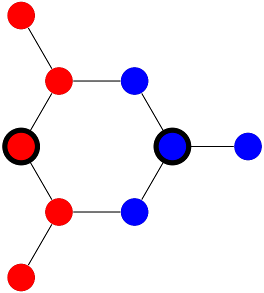

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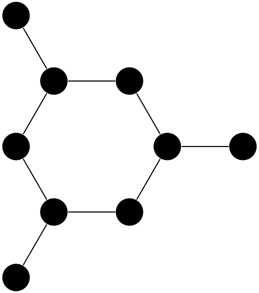

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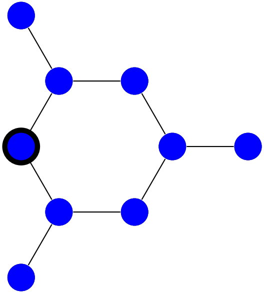

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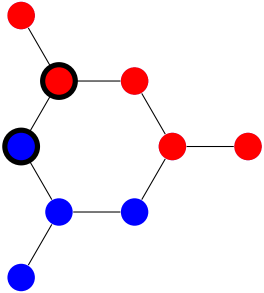

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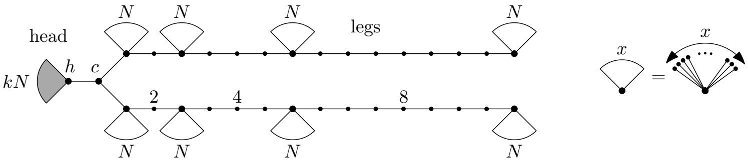

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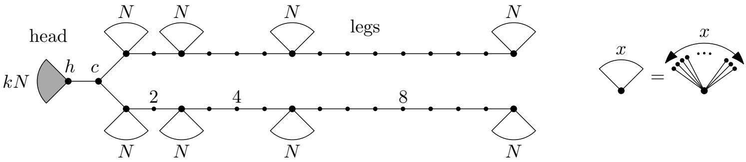

Optimal for $t = 2$.

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For every graph and t we have

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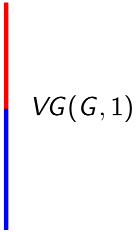

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Corollary

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Trees

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For every graph and t *we have* $\frac{1}{2}VR(G,1) \leq VR(G,t)$.

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Where is truth for $t > 2$ between $\frac{1}{4}$ and $\frac{1}{3}$?

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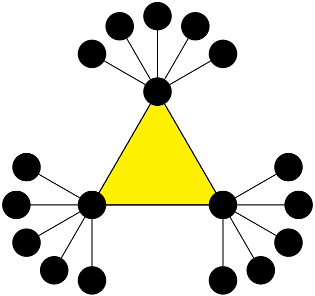

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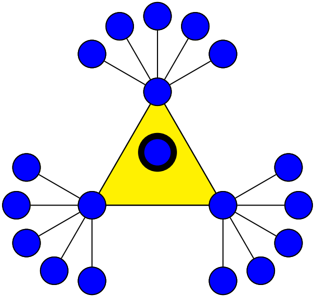

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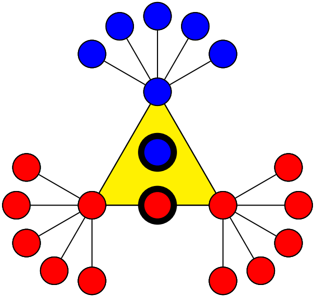

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if $t = 1$ then $VG(T, 1) \geq \frac{1}{2}$ sharp

if $t = 2$ then $VG(T, 2) > \frac{1}{3}$ sharp

if $t \geq 3$ then $\frac{1}{4} \leq VG(T, t)$ and $VG(T, t) < \frac{1}{3} + \epsilon$ possible.

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Is $VG(G, t : 1) < \epsilon$?

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Equivalent problem:

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$VG(G, t) < \epsilon$ possible for all t.
What if First makes t moves and Second makes 1?
Is $VG(G, t : 1) < \epsilon$?

Equivalent problem: Is there a finite function family, $\mathcal{F}$, such that for any $f_1, \ldots, f_t \in \mathcal{F}$ there is a $g \in \mathcal{F}$ such that $g > max(f_1, \ldots, f_t)$ on $(1 - \epsilon)$ fraction of inputs?

Thank you for your attention!